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Mispricing of the Black-Scholes-Merton Formula of Option Price when the Underlying Asset is distributed as a Bi-modal Distribution

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A B S T R A C T

This research proposes a new option pricing model. The model revises the unimodal probability distribution assumption used in the past, and proposes a bimodal probability distribution for option pricing. The bimodal probability distribution proposed in this study can be degenerated to a unimodal probability distribution under some special conditions. Such that, the option pricing model derived from the unimodal probability distributions will be a special extreme case of that estimated result of the model in this study. On the other hand, the bimodal probability distribution can be used to explain why the distribution has a fat-tail probability when some factors, such as the financial crisis, the trade war between the United States and China, the spread of the COVID-19 epidemic, etc., which continue to affect the price changes of the underlying asset written on options. In this situation, the distribution does not necessarily decrease gradually like the tail of the unimodal distribution; on the contrary, there will be another local mode. In the simulation calculations in this study, the traditional Black-Scholes-Merton model has a situation where the option price is incorrectly estimated (overestimated or underestimated) whenever the distribution of underlying asset's future prices is not unimodal. However, adding the assumption of bimodal probability distribution can properly explain this mis-estimation phenomenon and make corrections.

Keyword: Option pricing model, Bimodal distribution; Skewness parameter; Volatility
JEL: G13

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1. Introduction

Since the French mathematician Bachelier published his doctoral dissertation in 1900, titled as "Théorie de la Spéculation", his pioneering work has derived the option pricing theory through the use of Brownian motion stochastic process. In the important works of Black and Scholes (1973) and Merton (1973), they derive a closed-form formula for the price of option. Because of their landmark work, Merton and Scholes have won the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1997[2].

After that, numerous studies to test their option pricing model have been carried out. Besides the deviation from observed and theoretical option prices in these tests, the most stringent result is the calculation of an implied volatility smile (Chiras and Manaster, 1978; Rubinstein, 1994; Chen, Hung, and Hsiao, 2000; Hull, 2009; Corsi, Fusari, and La Vecchia, 2013). Because Black-Scholes-Merton formula assumes a constant volatility for the derivation of their option pricing formula, the existence of a volatility smile makes the empirical application of their formula questionable (Cox, Ingersoll, and Ross, 1985; Johnson and Shanno 1987; Rubinstein, 1994; Duan, 1995; Chen *et al*., 2000; Kim *et al*., 2009; Chen *et al*., 2012).

On the other hand, a challenge of the traditional Black-Scholes-Merton formula for valuing a European option is the distribution of future price of the underlying asset which is set to be a log-normal distribution. The log-normal distribution is unimodal and positive skew. However, as illustrated in Hull (2009), the probability distribution of future price of underlying asset may be a mixture distribution which combines two news, one is corresponding to favorable news, and the other is corresponding to unfavorable news. As a result, the distribution cannot be log-normal any more, instead, it will be a bimodal distribution.

Accordingly, by the spirit of De Schepper and Heijnen (2007), this study proposes a new family of distributions which are locally bimodal to correct the unimodal assumption of the traditional option pricing models. The second contribution of this study is to explain why the Black-Scholes-Merton model is mis-estimated. As shown in the Dang, Nguyen, and Sewell (2016) and Godin and Trottier (2021), the Black-Scholes-Merton model is over-estimated/under-estimated, however, they do not give a proper explanation to the results. Therefore, this study uses the parameters of volatility and degree of asymmetry of the bimodal distribution to explain the anomaly.

The structure of this study is structured as follows: the second chapter is to discuss the difference of option pricing models, the traditional Black-Scholes-Merton model and local bimodal distribution model. The third chapter shows the simulation results of the local bimodal distribution model and compare them with that predicted by the traditional Black-Scholes-

[2] Fischer Black died in 1995, and the Nobel Prize in Economics was awarded in 1997 to the other two scholars of the Black-Scholes-Merton formula, Myron Scholes and Robert Merton.

Merton. Some concluding remarks and suggestions are shown in the fourth chapter.

2. Option Pricing Models

2.1 Black-Scholes-Merton Model

It the assumption of Merton (1973), asset's future value (V) is followed as

$$dV_t = (\alpha \cdot V_t - C) \cdot dt + \sigma \cdot V_t \cdot dZ_t, \tag{1}$$

where α is the instantaneous expected rate of return on the asset; σ is the instantaneous standard deviation of the return on the asset; dZ_t is a standard Gauss-Wiener process.

Furthermore, assume that there is a call option with exercise price K written on the underlying asset S_t. As shown in Kwok (2008), its value will be given by

$$C_t = E_t\left[e^{-r\cdot(T-t)} \cdot \max(S_T - K, 0) \middle| \Im_t \right], \tag{2}$$

where, r is the discount rate, T is the maturity date, K is the exercise price, and $\Im_t$ is the information up to time t. If the future price of the underlying asset is distributed as a log-normal distribution, then by the Black-Scholes-Merton formula[3], we have,

$$C_t = S_0 \cdot N(d_1) - K \cdot e^{-r\cdot(T-t)} \cdot N(d_2), \tag{3}$$

where, $d_1 = \dfrac{\ln\left(\dfrac{S_0}{K}\right) + \left(r + \dfrac{\sigma^2}{2}\right)\cdot(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$, σ is the volatility of the price of the underlying asset. In addition, $N(\cdot)$ is the distribution function for a standard normal distribution, namely, $N(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz$.

Consider a European call written on some asset with exercise price $K = \$50$, which is matured $T - t = \frac{1}{12}$ (year). Moreover, suppose the discount rate ranges from 0% to 10% and volatility ranges from 0.01% to 40%, then the future payoff of the call option is given as follows:

$$\Pi_T(S_T, K) = \max(S_T - 50, 0), \tag{4}$$

and the discounted conditional expected payoff is

[3] Another version for the currency option is derived in Garman and Kohlhagen (1983).

$$C_t = E_t\left[e^{-\frac{2.5\%}{12}} \cdot \max(S_T - 50, 0) \middle| \Im_t \right], \tag{5}$$

where the distribution of $(S_T - 50)$ is distributed as a log-normal distribution. And the simulation results are shown in the following figure.

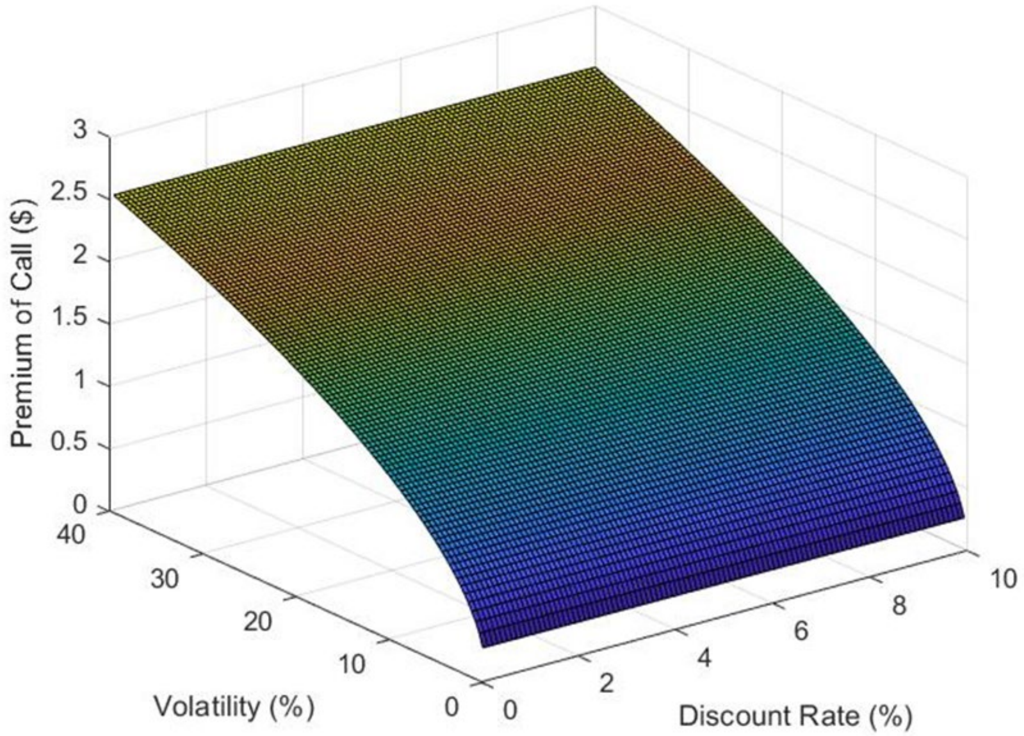


Figure 1. The premium of call by using Black-Scholes model

Figure 1 shows the simulated price of a European call option that expires in one month with an exercise price of $50 by using the log-normal distribution assumption. Since the log-normal distribution assumption is unimodal then the call's premium is increasing as the volatility increases.

2.2. Local Bimodal Distribution Model

In Borland (2002) and Borland and Bouchaud (2004), they propose a new idea that the distribution of underlying asset returns maybe non-Gaussian, which is different to the Black-Scholes-Merton model. Though Chalasani, Jha, and Saias (1999) and Aït-Sahalia and Duarte (2003) and Wilkens (2016) try to use a mixture distribution assumption to capture the non-Gaussian features of distribution of underlying asset's returns, however, their results show that the distributions of underlying asset's return are unimodal with skew and then thinner tail-probability (Jondeau, Poon, and Rockinger, 2007; Vellekoop and Nieuwenhuis, 2007; Markose and Alentorn, 2011; Markose and Alentorn, 2011).

However, as mentioned in Hull (2009), if the probability distribution of future price of underlying asset is a mixture distribution which combines two news, one is corresponding to favorable news, and the other is corresponding to unfavorable news, for instance, some macroeconomic factors shocks (Hutchison and Sushko, 2013), 1997 Southeast Asian financial crisis, 2008 U.S. subprime mortgage financial crisis, 2009 global financial tsunami, the US-China trade war since 2018, 2003 SARS outbreak, and COVID-19 pandemic (Hsiao, 2022), etc., such that, the distribution is no longer to be log-normal. It may consist of two facets: the difference of asset's favorable and unfavorable prices, the other is how often the favorable/unfavorable prices occur. Therefore, the premium of a European call option written on an asset with multi-modal distribution will never be computed by eq. (3), the Black-Sholes-Merton formula.

Hence, this study tries to use a local bimodal distribution to capture these situations of future price of underlying asset. As described in Hsiao (2021, 2022), the specification of a generalized local bimodal distribution is given as follows:

$$H(x;\delta_1,\delta_2,\lambda)=k(\delta_1,\delta_2,\sigma)\cdot\exp\left(-(x-\delta_1)^2-\frac{\lambda^2}{(x-\delta_2)^2}\right)\cdot I_{\{x\neq\delta_2\}}(x), \tag{6}$$

where, $I_{\{x\neq\delta_2\}}(\cdot)$ is an indicator function that gives value of 1 when $x\neq\delta_2$, and 0 when $x=\delta_2$. Moreover, $k(\delta_1,\delta_2,\lambda)$ is a positive function with parameters, δ_1, δ_2, and λ, such that,

$$\int_{-\infty}^{+\infty}H(x;\delta_1,\delta_2,\lambda)\,dx=1. \tag{7}$$

Hence, if the future price of the underlying asset is distributed as a local bimodal distribution, then the call's premium can be found according to eq. (2).

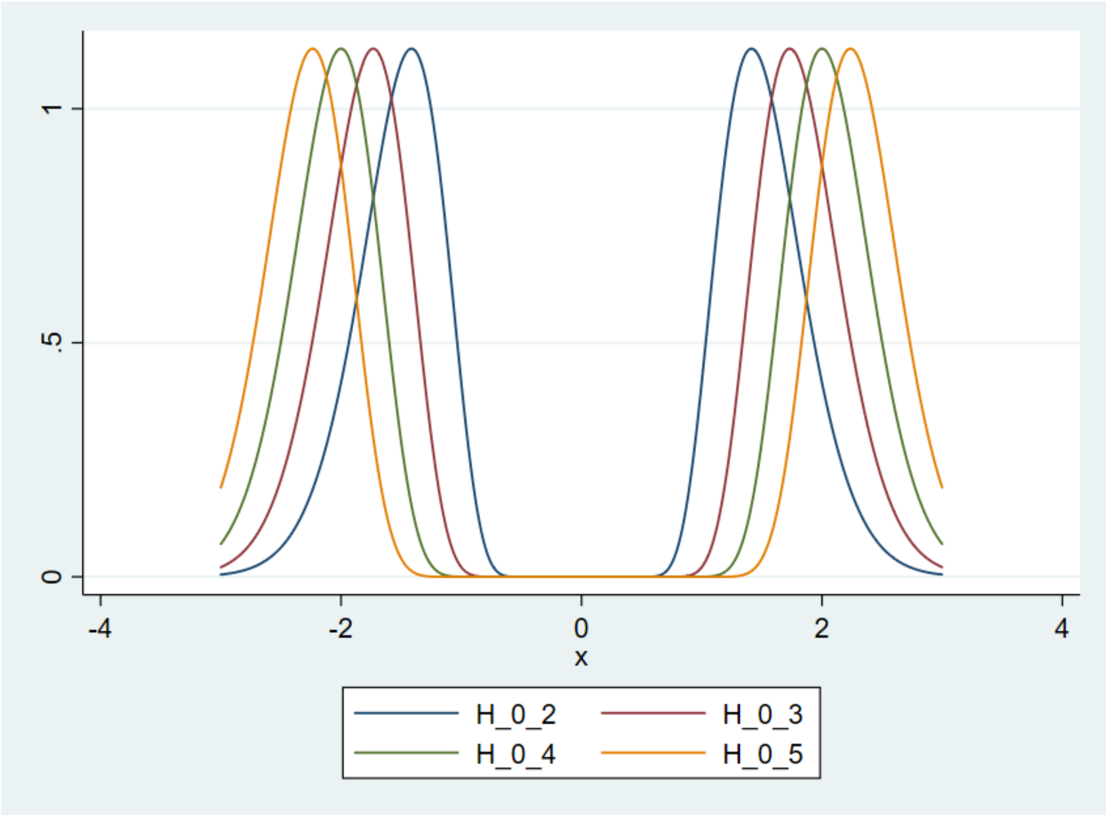
3. Simulation Results

3.1 Symmetric Case

As mentioned above, in the eq. (6), if the parameters δ_1 and δ_2 are zero, and λ is positive, then the distribution is given as

$$H_0(x;\lambda)=k(\lambda)\cdot\exp\left(-x^2-\frac{\lambda^2}{x^2}\right)\cdot I_{\{x\neq 0\}}(x), \tag{8}$$

with $k(\lambda)=\dfrac{\exp(2\lambda)}{\sqrt{\pi}}>0$. In this assumption, since $H_0(-x;\lambda)=H_0(x;\lambda)$, $\forall x\neq 0$, then the distribution is symmetric to the y-axis and its graph is shown in the following figure.



Note: H_0_2 (in blue) is for $\lambda=2$; H_0_3 (in red) is for $\lambda=3$; H_0_4 (in green) is for $\lambda=4$; H_0_5 (in orange) is for $\lambda=5$.

Figure 2. The graphs of the symmetric distribution with different values of λ

It can be seen that it is a bimodal distribution and the larger the value of λ, the longer the distance between the two modes. On the other hand, the smaller the value of λ, the thinner the tail probability.

For the same conditions of call, and given the distribution of (S_T-50) is $H_0(\cdot;\lambda)$. Such that, the simulation result of the call's premium is around \$0.4874, while it is around \$0.2389 by using the Black-Scholes-Merton model, given that $\lambda=36\%=\sigma$ and $r=2.5\%$. There is a significant difference between two approaches.

Furthermore, according to eq. (2), the simulation result by using the symmetric bimodal distribution for the different level of volatility and discount rates can be found in the following figure.

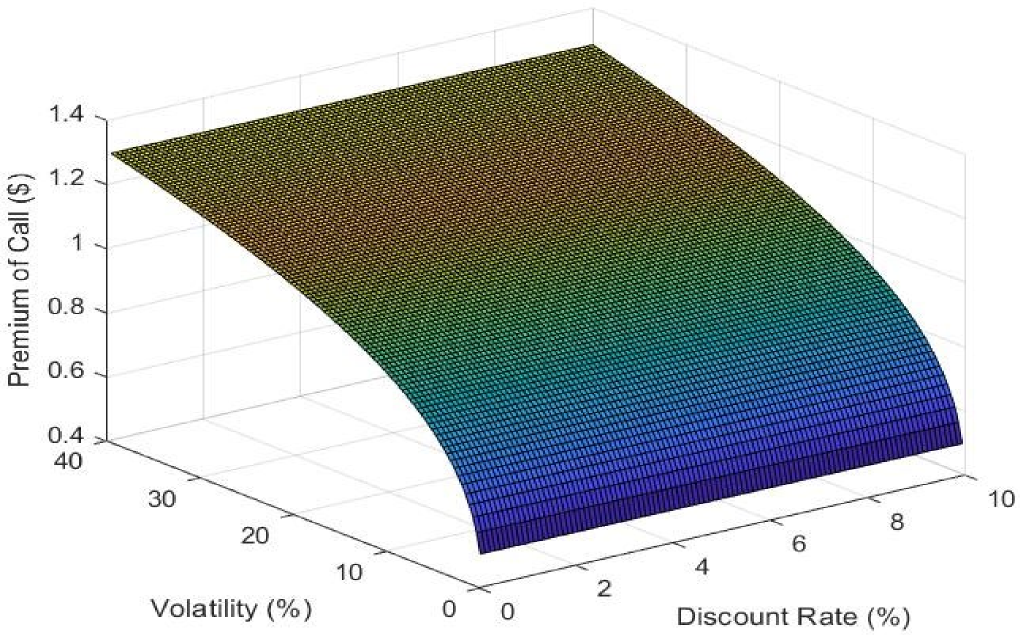
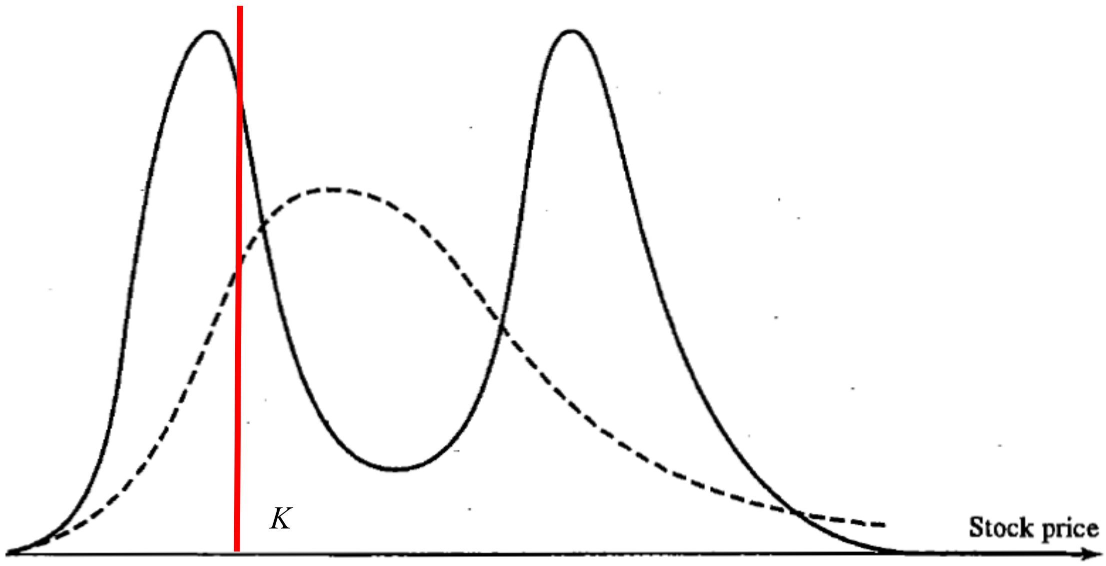


Figure 3. The premium of call by using symmetric bimodal distribution

Figure 3 shows the simulated price of a European call option that expires in one month with an exercise price of $50 by using the symmetric bimodal distribution. Compared to Figure 1, these two figures have the same pattern, that is, the more volatile (λ) of the asset's future price, the more valuable the European call and the lower the discount rate, the higher the European call.

Furthermore, the unimodal property of log-normal distribution assumes that the future price of the underlying asset will be only one possible price with the highest possibility. However, if there are some news is favorable and some is unfavorable, such that two more future prices will more likely happen than others. Hence, a larger value of λ represents that the price of underlying asset is more volatile since the distance between two prices (favorable and unfavorable) is relatively long. In these situations, the call should be less valuable than the traditional Black-Scholes model predicted. There are two reasons to explain this result. First, when some possible future prices are unfavorable to the call, that is, $S_T(\omega) < K$, for some state $\omega \in \Omega$, then the call's payoffs are zero in these states. As a result, the call is more likely to be out-of-money.

The second reason is the probabilities of the in-the-money call will less than a log-normal distribution, such that, the discounted expectation of the call's payoff will less than that computed by a log-normal distribution. For instance, as shown in the following figure, suppose that the solid line is the true distribution of future price for the underlying asset and the dashed line is the graph of the probability density function (*p.d.f.*) of a log-normal distribution. Then the expectation of payoff by using log-normal distribution is larger than that by using the true distribution. For the reasons mentioned above, the call's price predicted by Black-Scholes model will be over-estimated in the high volatility situation.



Source: Hull (2009).

Figure 4. Possible Distribution of Underlying Asset's Future Price.

Similarly, the Black-Scholes model will under-estimate the true value of the call in the lower volatility because of the positive skewness property of log-normal distribution. For a unimodal positive skew distribution, its mode, median, and mean have the following relationship:

$$Mode < Median < Mean\,. \tag{9}$$

Therefore, a positive skew probability density function is thinner of tail probabilities. And then the expectation using the log-normal distribution is less than that by using the true distribution which is not unimodal. As a result, the Black-Scholes model will under-estimate the true value of the call option. Accordingly, the following figure shows the difference of call's premium between that predicted by the traditional Black-Scholes model and symmetric bimodal distribution.

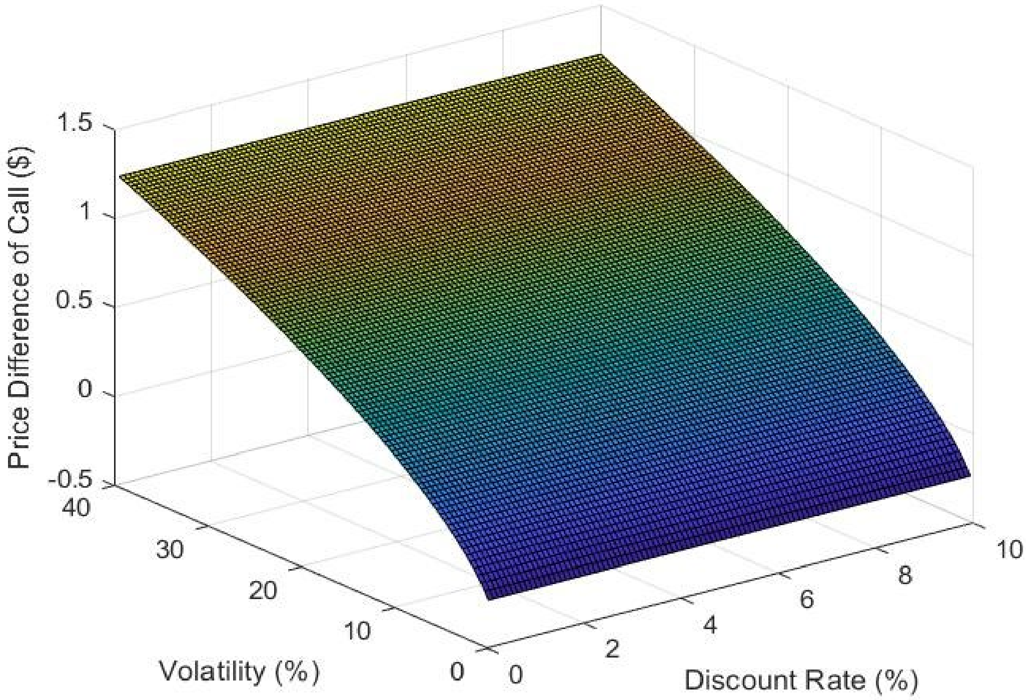


Figure 5. Price difference between BS model and symmetry distribution

The price difference is given by

$$\text{PriceDiff}_{BS-Sym} \equiv Call_{BS} - Call_{Symmtry}\,, \tag{10}$$

where $Call_{BS}$ and $Call_{Symmtry}$ represent the call's premium predicted by Black-Scholes model and symmetric bimodal distribution, respectively. As shown in Figure 4, in the high volatility situation, the traditional Black-Scholes model over-estimates the premium of call and under-estimates in the low volatility situation.

3.2 Asymmetric Cases

Whenever δ_1 or δ_2 is non-zero, then the function of eq. (6) is not symmetric to any vertical line. And if $\delta_1 = \delta \neq 0$ and $\delta_2 = 0$, then the function of eq. (6) will be negative skew; on the other hand, if $\delta_1 = 0$ and $\delta_2 = \delta \neq 0$, then the function of eq. (6) will be positive skew.

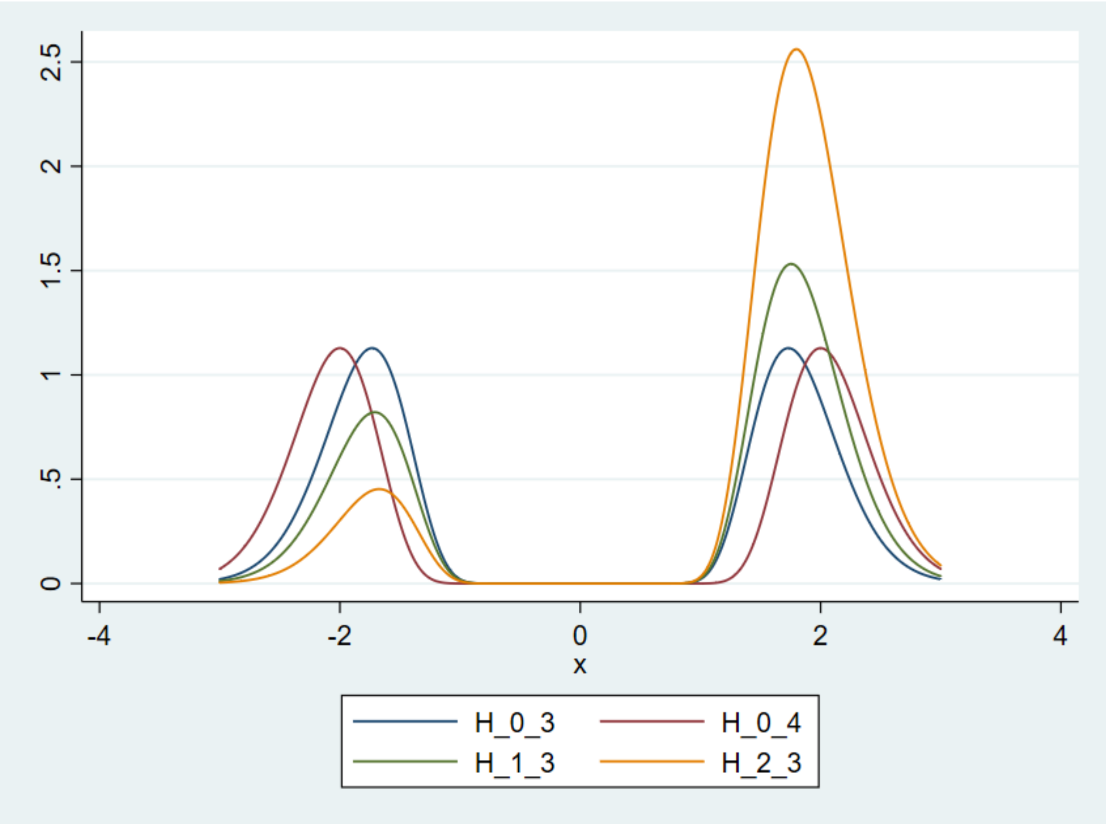
3.2.1 Negative Skew Case

Given the function

$$H_{NS}(x;\delta,\lambda) = k_{NS}(\delta,\lambda)\cdot \exp\left(-(x-\delta)^2 - \frac{\lambda^2}{x^2}\right)\cdot I_{\{x\neq 0\}}(x)\,, \tag{11}$$

with $k_{NS}(\delta,\lambda)>0$, such that, $\int_{-\infty}^{\infty} H_{NS}(x;\delta,\lambda)\,dx=1$.

In this assumption, since $H_{NS}(-x;\delta,\lambda)\neq H_{NS}(x;\delta,\lambda)$, $\forall x\neq 0$, then the distribution is asymmetric. The graph is shown in the following figure.



Note: H_0_3 (in blue) is a benchmark of the symmetry distribution with $\delta=0.0$ and $\lambda=3$; H_1_3 (in red) is asymmetric with $\delta=0.25$ and $\lambda=3$. H_2_3 (in orange) is asymmetric with $\delta=0.5$ and $\lambda=3$.

Figure 5. Family of negative skew distributions

In the Figure 4, we can find that each curve has two local modes, however, the relative extreme value of the curves are unequal, and all the functions are negative skew. Moreover, compared to the symmetric cases (in blue and red, respectively), the larger the value of λ, the longer the distance between two relative maxima.

Consider the same conditions of call, and if the distribution of (S_T-50) is $H_{NS}(\cdot;\delta,\lambda)$, with $\delta=$ 1.0 and $\lambda\in(0.0\%,40\%)$, then the simulation result by using the local bimodal distribution can be find in the following figure.

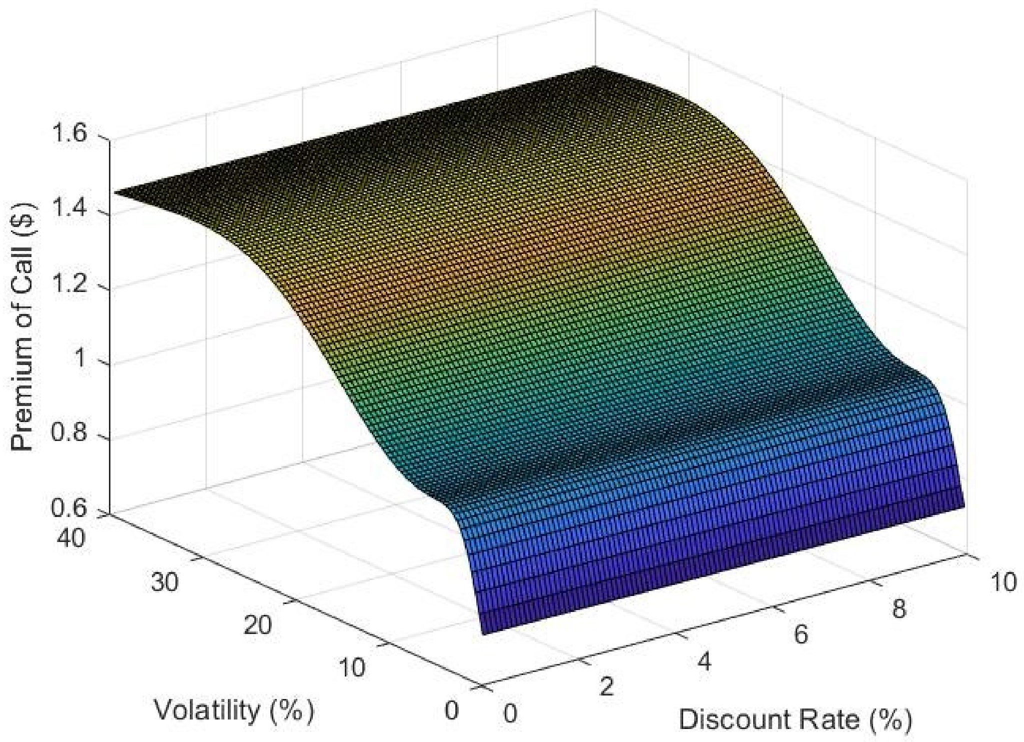


Figure 6. The premium of call by using negative skew bimodal distribution

In Figure 6, the more volatile of the future price of the underlying asset, the more valuable of the European call. However, different to Figure 1 and 3, the surface in Figure 6 is not so smooth. In situation of high volatility, the premium of the European call option is concave downward, that is, given a certain level of discount rate, there exists a $\lambda^* \geq \bar{\lambda} > 0$, such that, $C_t(\lambda^*, r) \geq C_t(\lambda, r)$, for $\lambda \geq \bar{\lambda}$. And it is relatively flat when the volatility of the future price of the underlying asset falls between about 10% and 20%, no matter what level of discount rates. Moreover, when the volatility is less than 5%, the value of the European call drop sharply.

Next, consider the effect of the difference of two local modes on the value of call. As shown in Figure 5, when the value of δ is greater, the difference between the heights of the two local modes is greater. That is to say, the probability of occurrence of the high price on the right (favorable) is much higher than the high price on the left (unfavorable). The value of λ is mainly determines the distance between the two. The following figure shows the relationship between the value of λ and δ and the value of call.

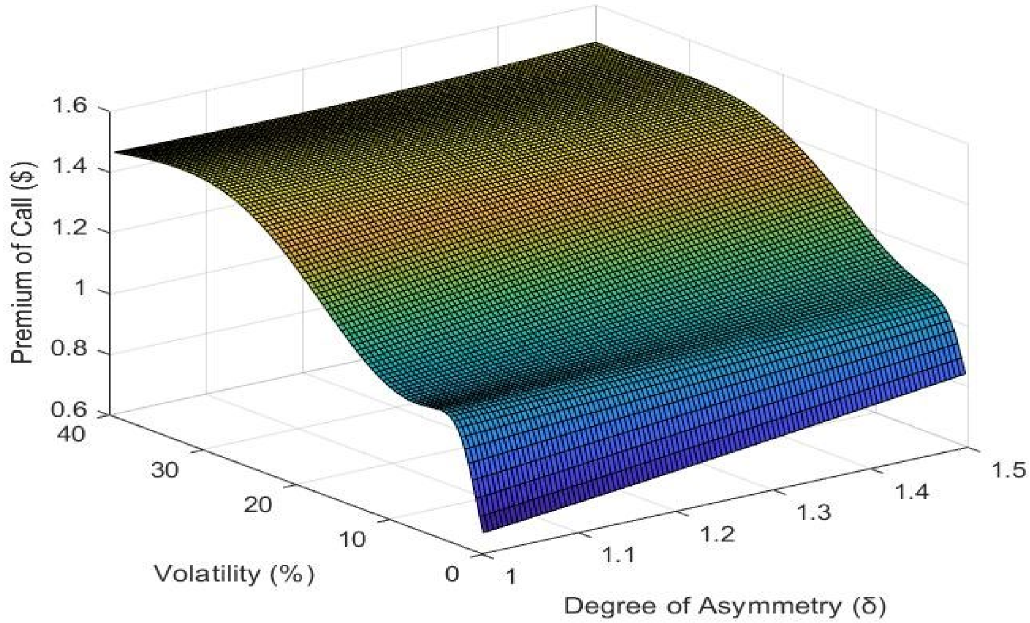


Figure 7. The Effect of δ and λ on the premium of call by using $H_{NS}(\cdot;\delta,\lambda)$

As shown in Figure 7, at the low level of volatility ($\lambda < 5\%$), the premium of call is increasing in the value of *δ*. And in the high level of volatility ($\lambda > 30\%$), the premium of call is "smile", that is, either a lower value of *δ* or higher one, the premium of the call is higher than that in the middle range of *δ*. As a result, there exists $\delta^*(\lambda) > 0$, such that, $C_t(\delta^*(\lambda), \lambda) \le C_t(\delta, \lambda)$, for a given $\lambda > 0$ .On the other hand, there is concave downward of the call's premium in the high level of volatility. This result indicates that for a given *δ*, there exists a $\lambda^*(\delta) \ge \overline{\lambda}(\delta)$, such that, $C_t(\delta, \lambda^*(\delta)) \ge C_t(\delta, \overline{\lambda}(\delta))$.

These results are different to that predicted the traditional Black-Scholes-Merton model because of the features of the local bimodal distribution. First, since the value of *λ*, a volatility measure, determines distance between the two modes. Such that, a large value of *λ* stands for the position of unfavorable price is more far away to the exercise price (*K*) than that of the favorable price. Under this situation, the discounted expectation values of call's payoff will be smaller than that in the middle range of volatility, for a given value of *δ*. Second, the value of *δ* determines the difference of probabilities of the two modes. Namely, by the specification of negative skew distribution, the larger value of *δ*, the more difference of the probabilities of favorable price and unfavorable price. Such that, it means that the discounted expectation value of call's payoff in a large value of *δ* will be higher than that in small value of *δ*.

Accordingly, the following figure shows the difference of call's premium between that predicted by the traditional Black-Scholes model and negative skew local bimodal distribution.

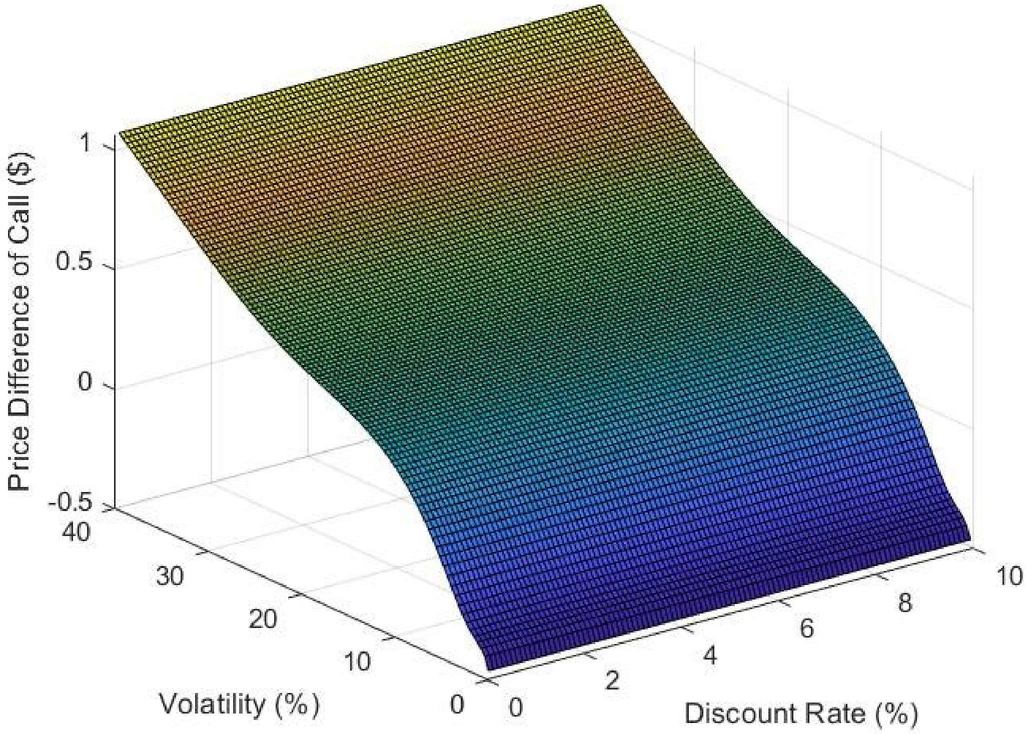


Figure 8. Price difference between BSM model and negative skew distribution

The price difference is given by

$$\text{PriceDiff}_{BS-NS}(\lambda,\sigma,r) \equiv Call_{BS} - Call_{NS}, \tag{12}$$

where $Call_{NS}$ represents the call's premium predicted by negative skew local bimodal distribution. As shown in Figure 8, in the high volatility situation, the traditional Black-Scholes model over-estimates the premium of call and under-estimates in the low volatility situation.

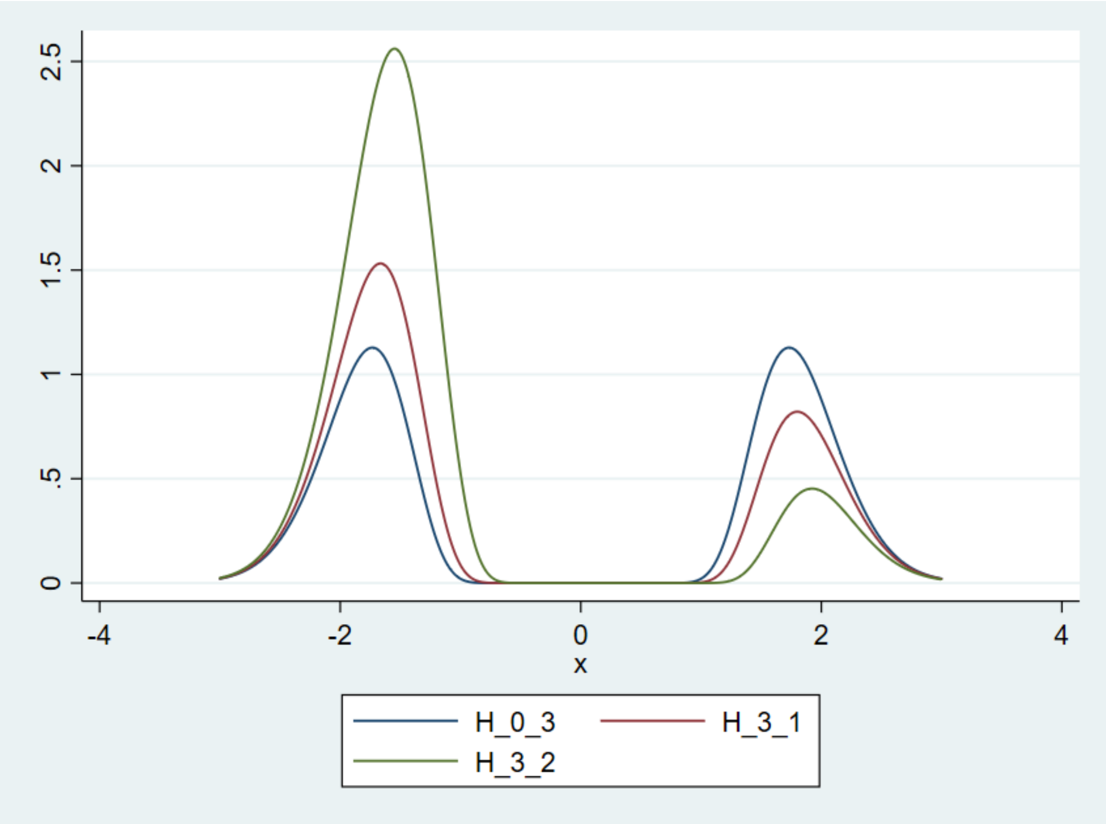
3.2.2 Positive Skew Case

Next, consider the following family of functions that are positive skew:

$$H_{PS}(x;\delta,\lambda) = k_{PS}(\delta,\lambda)\cdot \exp\left(-x^2 - \frac{\lambda^2}{(x-\delta)^2}\right)\cdot I_{\{x\neq\delta\}}(x), \tag{13}$$

with $k_{PS}(\delta,\lambda) > 0$, such that, $\int_{-\infty}^{\infty} H_{PS}(x;\delta,\lambda)\,dx = 1$.

In this assumption, since $H_{PS}(-x;\delta,\lambda) \neq H_{PS}(x;\delta,\lambda)$, $\forall x \neq 0$, then the distribution is asymmetric. The graph is shown in the following figure.



Note: H_0_3 (in blue) is a benchmark of the symmetry distribution with $\delta = 0.0$ and $\lambda = 3$; H_3_1 (in red) is asymmetric with $\delta = 0.25$ and $\lambda = 3$; H_3_2 (in green) is asymmetric with $\delta = 0.5$ and $\lambda = 3$.

Figure 9. Family of positive skew distributions

In the Figure 7, we can find that each curve has two local modes, however, the relative extreme value of the curves are unequal. Moreover, all the functions are positive skew and the larger the value of λ, the longer the distance between two relative maxima.

Furthermore, consider the same conditions of the call, and if $(S_T - 50)$ is distributed as $H_{PS}(\cdot;\delta,\lambda)$, with $\delta \in [1.0, 1.5]$ and $\lambda \in (0.0\%, 40\%)$. Such that, according to the eq. (2), the simulation result by using the local bimodal distribution can be find in the following figure.

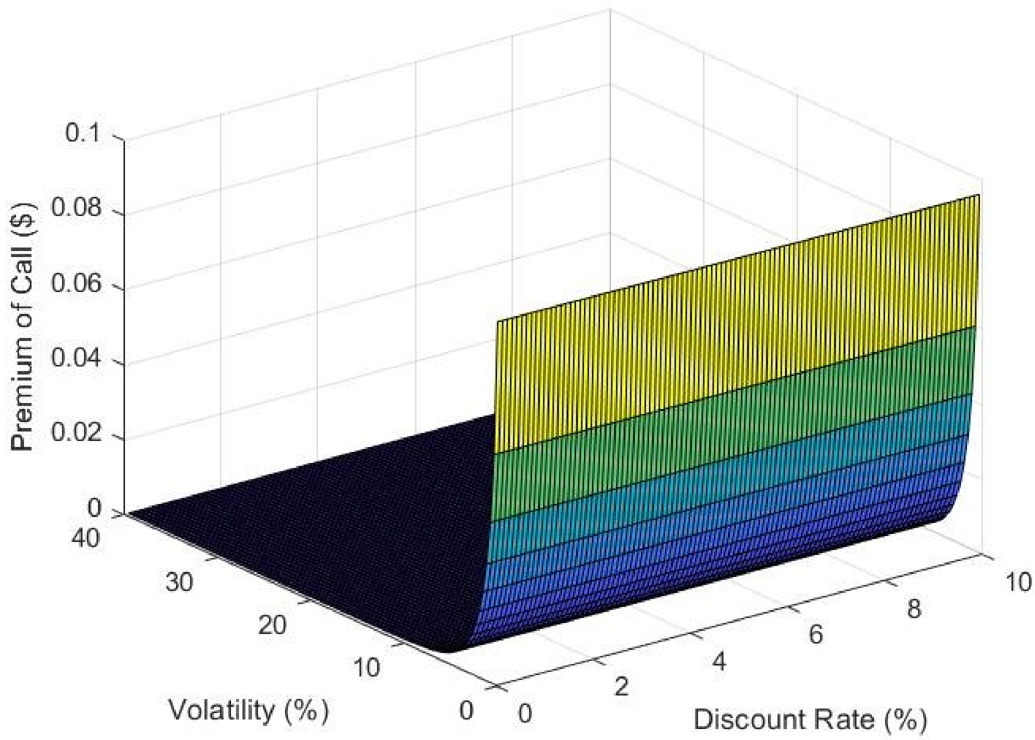


Figure 10. The premium of call by using positive skew bimodal distribution

In Figure 10, the more volatile of the future price of the underlying asset, the less valuable of the European call. This is contradict to the original option pricing theorems. This result is because when the volatility is higher, the distance between the two modes is farther, and that is, the distance between the two highest frequency possible future prices (favorable/unfavorable) is farther. Though the unfavorable price is far way to the favorable price, In other words, the less the probability of the in-the-money part, the lower the expected value of call's future payoff. This situation also leads to the lower value of the European call option.

Next, consider the effect of the difference of two local modes on the value of call. As shown in Figure 9, when the value of δ is greater, the difference between the heights of the two local modes is greater. That is to say, the probability of occurrence of the high price on the left (unfavorable) is much higher than the high price on the right (favorable). The value of λ is mainly determines the distance between the two modes. The following figure shows the relationship between the value of λ and δ and the value of call.

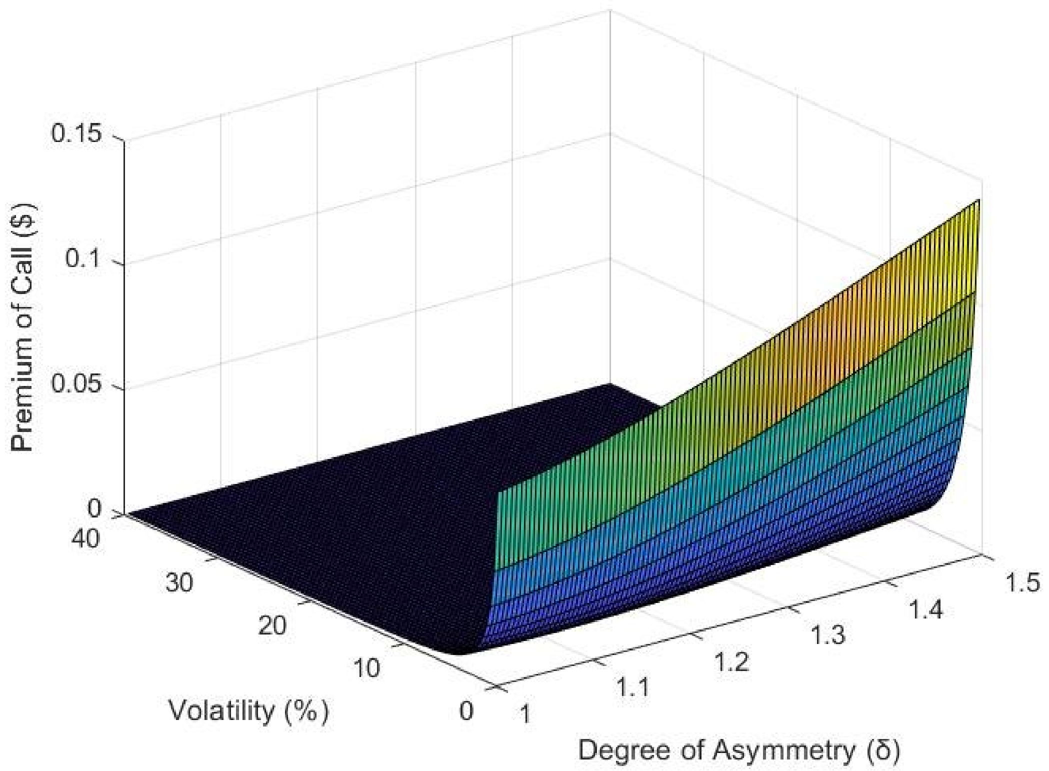


Figure 11. The Effect of δ and λ on the premium of call by using $H_{PS}(\cdot;\delta,\lambda)$

In Figure 11, it shows the same feature as the results in Figure 10, which a lower volatility of future prices of the underlying asset will make the call more valuable. It also can be found that given a low level of volatility, the call's premium is increasing in the value of δ. These results are different to that predicted the traditional Black-Scholes-Merton model because of the features of the local bimodal distribution. First, since the value of λ determines the distance between the two modes. Such that, a large value of λ stands for the position of favorable price is more far away to the exercise price (K) than that of the unfavorable price. Hence, for a given value of δ, the discounted expectation values of call's payoff will be increasing as the volatility decreases. Second, the value of δ determines the difference of probabilities of the two modes. Namely, by the specification of positive skew distribution, the larger value of δ, the more difference of the probabilities of unfavorable price and favorable price. Meanwhile, it also stands for that the probability of occurrence of the favorable price is still higher than the other unimodal distributions, such as log-normal, exponential, or others. Such that, the discounted expectation value of call's payoff in a large value of δ will be higher than that in small value of δ.

To compare the estimating efficiency, the price difference is given by

$$\text{PriceDiff}_{BS-PS} \equiv Call_{BS} - Call_{PS}, \tag{14}$$

where $Call_{PS}$ represents the call's premium predicted by positive skew local bimodal distribution. Accordingly, the following figure shows the difference of call's premium between that predicted by the traditional Black-Scholes model and negative skew local bimodal distribution.

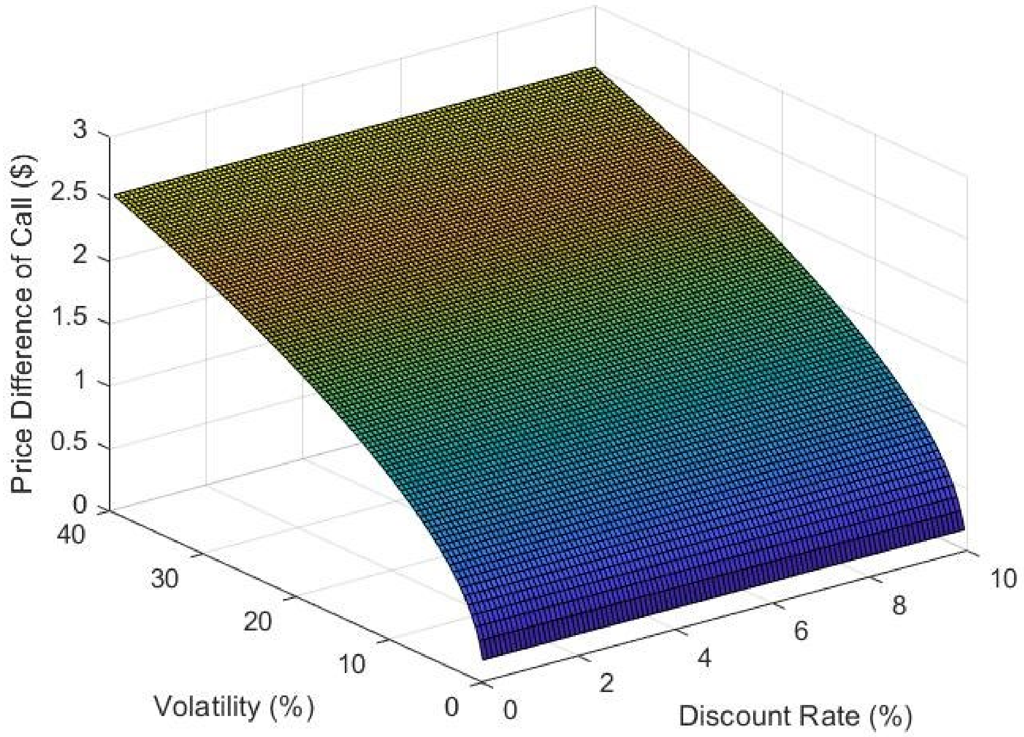


Figure 12. Price difference between BS model and positive skew distribution

As shown in Figure 12, the traditional Black-Scholes model over-estimates the premium of call in all volatility situation. This is because in the Black-Scholes model assumption, the log-normal distribution is a unimodal distribution, and the tail probability on the right side will be relatively lower than the positive skew probability density functions. Therefore, the in-the-money part of the call is assigned too much probability by the Black-Scholes model, which causes the estimated European call to be too high. It means that the Black-Scholes model overestimates the price of the European call. And as the volatility decreasing, the over-estimation of the Black-Scholes model decreases.

4. Conclusions

This study first proposes a local bimodal probability distribution family to describe the future price changes of the underlying asset written on a European call. Although this setting is different from the assumptions of the traditional Black-Scholes-Merton model, the log-normality assumption is a degenerated case of the local bimodal distribution as λ approaches to zero or δ approaches to infinity for the asymmetric cases.

Furthermore, the simulation results in this study show that when the price volatility increases, the price of the option will not necessarily increase. When the price of the underlying asset is a bimodal symmetrical distribution, as estimated by the Black-Scholes-Merton model, the higher the volatility, the higher the option price. However, when the volatility is high, the Black-Scholes-Merton model will overestimate the premium of the call; and when the volatility is low, the Black-Scholes model will underestimate.

When the price of the underlying asset is a bimodal asymmetric probability distribution, the results of positive and negative skew are inconsistent. First, when the probability distribution is negative skew, the call price changes with the degree of asymmetry. And given a degree of volatility, it presents a smiling curve. Second, when the probability distribution is positive skewness, the call option price increases as the degree of asymmetry increases. And when the volatility becomes lower, the call option is more valuable, but the higher the volatility, the less valuable the call. This result is different from the results estimated by various options pricing models in the past.

The probability model proposed in this study is closer to the actual probability distribution of the underlying asset. Therefore, the simulation results can be used to correct the current bias in the estimation of the option price, and can even be used as a reference price when the option is issued.

Reference

Aït-Sahalia, Y., and Duarte, J. (2003). Nonparametric Option Pricing under Shape Restrictions. *Journal of Econometrics*, 116(1-2), 9-47.

Bachelier, L. (1900). Théorie de la Spéculation. Annales scientifiques de l'École Normale Sup érieure, 3(17), 21-86.

Black, F., and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637-659.

Borland, L. (2002). A Theory of Non-Gaussian Option Pricing. *Quantitative Finance*, 2(6), 415-431.

Borland, L., and Bouchaud, J. P. (2004). A Non-Gaussian Option Pricing Model with Skew. *Quantitative Finance*, 4(5), 499-514.

Chalasani, P., Jha, S., and Saias, I. (1999). Approximate Option Pricing. *Algorithmica*, 25(1), 2-21.

Chen, H., Hung, M. W., and Hsiao, C. M. (2000). Kernel Weighted GMM Estimation for Term Structure of Continuous-Time Interest Rates. Unpublished Working paper.

Chen, Y. C., Lyuu, Y. D., and Wen, K. W. (2012). The Complexity of GARCH Option Pricing Models. *Journal of Information Science and Engineering*, 28(4), 689-704.

Chiras, D. P., and Manaster, S. (1978). The Information Content of Option Prices and a Test of Market Efficiency. *Journal of Financial Economics*, 6(2-3), 213-234.

Corsi, F., Fusari, N., and La Vecchia, D. (2013). Realizing Smiles: Options Pricing with Realized Volatility. *Journal of Financial Economics*, 107(2), 284-304.

Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). A Theory of the Term Structure of Interest Rates. *Econometrica* 53 (2), 385-407.

Dang, D. M., Nguyen, D., and Sewell, G. (2016). Numerical Schemes for Pricing Asian Options under State-Dependent Regime-Switching Jump-Diffusion Models. *Computers and Mathematics with Applications*, 71(1), 443-458.

De Schepper, A., and Heijnen, B. (2007). Distribution-Free Option Pricing. *Insurance: Mathematics and Economics*, 40(2), 179-199.

Duan, J. C. (1995). The GARCH Option Pricing Model. *Mathematical Finance*, 5(1), 13-32.

Godin, F., and Trottier, D. A. (2021). Option Pricing in Regime-Switching Frameworks with the Extended Girsanov Principle. *Insurance*: *Mathematics and Economics*, 99(1), 116-129,

Garman, M. B., and Kohlhagen, S. W. (1983). Foreign Currency Option Values. *Journal of International Money and Finance*, 2(3), 231-237.

Hsiao, C. M. (2021). Nash Equilibrium, Heterogeneous Preferences and the Extended Hotelling Linear City. 2021 Taiwan Economic Association Annual Conference, 2021/12/11, Soochow University, Taipei, Taiwan.

Hsiao, C. M. (2022). Effects on Second Waves of COVID-19 Epidemics: Social Stringency, Economic Forces, and Public Health. *Theoretical Economics Letters*, 12(1), 278-320.

Hull, J. (2009). *Options, Futures, and Other Derivatives*, 7th Edition. Pearson Education International, Chapter 18.

Hutchison, M., and Sushko, V. (2013). Impact of Macro-economic Surprises on Carry Trade Activity. *Journal of Banking & Finance*, 37, 1133-1147.

Johnson, H., and Shanno, D. (1987). Option Pricing when the Variance is Changing. *Journal of Financial and Quantitative Analysis*, 22(2), 143-151.

Jondeau, E., Poon, S. H., and Rockinger, M. (2007). *Financial Modeling under Non-Gaussian Distributions*. Springer-Verlag London, Chapter 10.

Kim, Y. S., Rachev, S. T., Chung, D. M., and Bianchi, M. L. (2009). The Modified Tempered Stable Distribution GARCH Models and Option Pricing. *Probability and Mathematical Statistics*, 29(1), 91-117.

Kwok, Y. K. (2008). *Mathematical Models of Financial Derivatives*, 2nd Edition, Springer Berlin Heidelberg, German. Chapter 3.

Markose, S., and Alentorn, A. (2011). The Generalized Extreme Value Distribution, Implied Tail Index, and Option Pricing. *Journal of Derivatives*, 18(3), 35-60.

Merton, R. C. (1973). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science*, 4(1), 141-183.

Rubinstein, M. (1994). Implied Binomial Trees. *Journal of Finance*, 49 (3), 771-818.

Vellekoop, M., and Nieuwenhuis, H. (2007). On Option Pricing Models in the Presence of Heavy Tails. *Quantitative Finance*, 7(5), 563-573.

Wilkens S. (2016). Option Pricing Based on Mixtures of Distributions: Evidence from the Eurex Index and Interest Rate Futures Options Market. In: Satchell S. (eds) *Derivatives and Hedge Funds*. Palgrave Macmillan, London. Chapter 18.